SLICOT Drives Tractors![1]

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Abstract

We describe the successful application of a SLICOT subroutine in a control engineering problem. Based on GPS data it is possible to automatically steer farm vehicles along a prescribed trajectory. The bottleneck for the successful on-line implementation of a LQG regulator is the numerical solution of a discrete-time algebraic Riccati equation in real-time and at high accuracy. This is achieved employing a Fortran 77 subroutine from the Subroutine Library in Control Theory — SLICOT.

1 Motivation

In control engineering applications, the numerical solution of one of the fundamental matrix equations of linear control theory often causes a bottleneck for its successful realization. Here we will describe one such application, the automatic steering of a farm tractor, in which this situation occurs. By using a SLICOT subroutine, the problem is resolved, i.e., the used subroutine yields a numerical solution with sufficient accuracy and in due time.

Through the development of ever more cheap and reliable GPS (Global Positioning System) receivers, automatic steering of ground vehicles along prescribed trajectories has become an attractive research topic for several reasons: hazardous operations can be performed without risking human life, cars may be automatically piloted to the desired destination ("smart highways" – an ongoing research project for several decades), maneuvering at high precision, etc. One of the first fields of application are vehicles used for farming on the large fields of Northern America. As farm vehicles usually move at moderate speed and agricultural fields usually are ideal in order to operate GPS properly as nothing blocks the GPS signals, they are among the first ones for which such a technology becomes feasible for industrial application.

We will describe a project realized by the GPS Lab at the Department of Aeronautics and Astronautics of Stanford University (USA). The project has been supported by Deere and Company and it is hoped that John Deere tractors equipped with the developed technology will soon go into production at Deere and Company.

The research team at Stanford has realized a control mechanism for automatic steering of a farm tractor. Though on first glimpse, this may not seem to be of major interest, there are several reasons why such a system may be desirable. On the large fields in the Midwest of the USA or Canada, tractor driving gets extremely boring. It is reported that drivers even fall asleep, causing significant damage by getting off track and thereby destroying many rows of crops or hoses for watering the fields. Even drivers awake may not be able to drive at necessary precision level to avoid crossing the watering hoses. As other trajectories than "back-and-forth, using U-turns" are difficult for manual drivers, automatic steering would make it possible to steer optimal patterns (e.g., spiral patterns). GPS–based controllers might enable a single driver to operate a convoy of several tractors at the same time or it could allow farmers to operate during the night, through heavy dust or fog. While a tractor driving automatically could be controlled from an office, there are also other reasons to free the farmer from driving. On large fields usually the soil conditions and other factors vary. It is therefore desirable to adapt bedding, seeding, fertilizing, cultivating, and irrigation to these variations and thereby minimizing the use of herbicides and pesticides while maximizing the crop. The manual adaption of these tasks is facilitated if the farm vehicle is steered automatically. Eventually, these adaptions may even be executed automatically as well.

2 Ground position determination

The approach taken in this project is to first measure the field and record every obstacles to be avoided like, e.g., watering devices, and then to program the tractor to follow a prescribed path. As deviations from the path are unavoidable, a regulator using reference tracking needs to be implemented in order to keep the tractor "on track". Therefore the knowledge of the exact location of the driving tractor is required to regulate the driven trajectory. This has become possible by using GPS.

The ***G**lobal **P**ositioning **S**ystem*[1] is a constellation of 24 satellites circling the Earth every 12 hours. Using standard geodetic methods it is possible to locate your position on earth by receiving the signals (position and time) of at least three (in theory — in practise at least four are necessary) satellites [15]. Since this system has been made available for civilian use it has found many applications from aviation to wilderness expeditions. Hand-held GPS receivers are now even available for outdoor camping. A precision of about 100 yards as attained by normal civilian GPS receivers is of course not sufficient for the application described here. Therefore, a high precision GPS-based system, called *Carrier Phase Differential GPS* (CDGPS), has been used here [7, 8, 10, 14]. CDGPS requires ground based local transmitters, called *pseudolites*. With this technology, it is possible to obtain position and attitude at centimeter-level and 0.1° accuracy. The technology has been made feasible for use in farming by reducing the number of necessary pseudolites for initializing the CDGPS to only one [13].

The research group at Stanford University has equipped the John Deere 7800 farm tractor shown in Figure 1[2] with such a CDGPS receiver. This has then been used for system identification and automatic control of the tractor [3, 7, 8, 11, 12, 14]. Here we will focus on the automatic control aspect. We will describe the model used to to design the controller for the tractor in the next section.

Figure 1: GPS–equipped tractor.

[1]The system is operated by the U.S. Department of Defense.

[2]Taken from [12].

3 Automatic control of a farm tractor

The state-space model used for simulating the tractor movement is derived from the nonlinear equations of motion; see [3, 10]. The relevant variables are shown in Figure 2[3]. The desired trajectory is part of the prescribed path to be followed by the tractor; d is the distance from the center of the rear axle of the tractor (which is the reference point) to the closest point on the reference trajectory, ψ is the yaw angle of the tractor relative to the desired trajectory, δ is the front wheel angle relative to the centerline of the tractor, and l is the distance from the front wheels to the reference point. Linearization of the equations of motion and removing

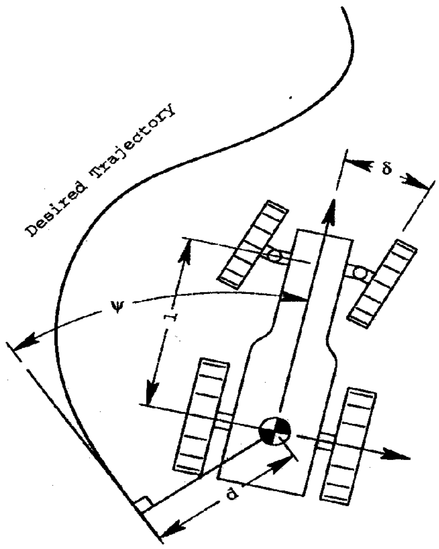

Figure 2: State space variables.

redundancies yields a continuous, fifth-order linear time-invariant (LTI) system in state-space form,

$$\begin{aligned} \dot{x} &= Ax + Bu, & (1)\\ y &= Cx, & (2) \end{aligned}$$

where the states are $x = [\psi,\, \dot{\psi},\, \delta,\, \dot{\delta},\, d]^T$ and

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & -\frac{1}{\tau_\psi} & \frac{V}{l\tau_\psi} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_u} & 0 \\ V & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\tau_u} \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

[3]Taken from [3].

Here, V is the forward velocity of the tractor while τ_ψ and τ_u are parameters which where identified using experimental data. The specification of trajectories and generation of reference states is described in detail in [3].

For the closed-loop control of the tractor, a discrete-time linear-quadratic Gaussian (LQG) regulator is used consisting of a linear-quadratic optimal control with reference state tracking [1] and a Kalman filter for estimating the states [4]. The disturbances (sensor noise/process noise) were modeled based on experimental data. The conversion between the continuous plant dynamics and GPS on the one side and the discrete LQG regulator on the other side was performed assuming zero-order hold on the inputs. The block diagram of the closed-loop system is shown in Figure 3[4].

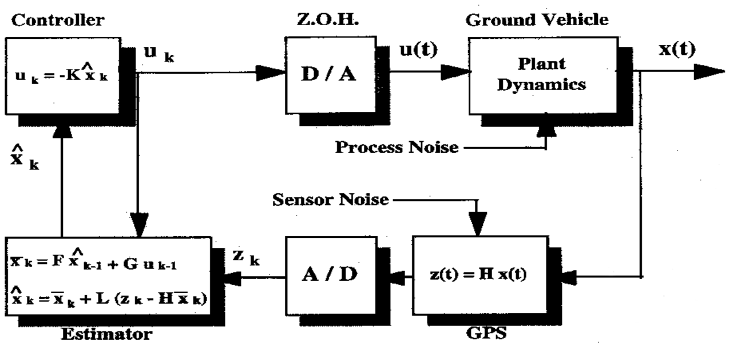

Figure 3: Block diagram of LQG regulator.

The control gains are obtained using the standard Riccati approach, i.e., via the stabilizing solution of the discrete-time algebraic Riccati equation (DARE)

$$X = C^TQC + A^TXA - A^TXB(R + B^TXB)^{-1}B^TXA, \tag{3}$$

where A, B, C are as in (1)–(2) while Q and R are the weighting matrices for state deviations from nominal and control inputs, respectively, in the quadratic cost functional to be minimized (see, e.g., [1, 4]). For satisfactory regulation results, a DARE of the form (3) has to be solved to high accuracy five times per second. This turns out to be the bottleneck for the realization of an automatic controlled farm tractor as all other necessary computations (matrix adds and multiplies, solution of linear systems, etc.) can be performed sufficiently fast on the available hardware. (The tractor is equipped with a 100MHz Pentium–PC.)

In a first approach the control gains were calculated with MATLAB[5] using gain-scheduling on forward velocity. The computed gains were uploaded to the computer on board of the tractor.

[4]Taken from [14].
[5]MATLAB is a registered trademark of The MathWorks, Inc.

The controller then switched between various gains based on the velocity V. Employing the SLICOT subroutine **SB02MD**[6] which provides an implementation of the Schur vector method for solving (3) (see [9]), it is now possible to compute the control gains on-line. The DARE is solved in roughly 5% of the 200msec sampling time which gives plenty of time for doing the other necessary calculations. Having the ability to solve the DARE in real time now means that one can use information from an on-line adaptive identification algorithm to actually improve the control going along. Before **SB02MD** was used, a fixed model had to be assumed and changing conditions could not be incorporated into the control law [2].

4 Concluding remarks

The GPS-based control strategy described in this article can be used for other land vehicles, and, using a slightly more complex CDGPS even for autolanding of planes; see, e.g., [5, 6].

The application described in this article demonstrates how the use of robust numerical software provided in SLICOT can enable control engineers to realize innovative ideas and to make them available for production processes.

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