# INTEGRATED MODEL-REDUCTION FACILITIES IN THE COMPUTATIONAL CONTROL DESIGN ENVIRONMENT ANDECS

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**Abstract.** The paper presents the new integrated model-reduction facilities available in the control engineering design environment ANDECS. The suite of interactive model reduction modules is based on a new generation of numerically reliable algorithms to solve model reduction and associated problems. The model reduction tools complement the already existing system analysis and design tools in ANDECS and strongly benefit of the advanced simulation and visualization aids, and the multi-objective programming facilities for parametric studies already available there.

**Keywords.** Model reduction, computer aided control system design, numerical methods, numerical software.

## 1. INTRODUCTION

Model reduction techniques serve two important tasks in solving control engineering problems. First, a *model order reduction* is frequently necessary to bring control design problems to a manageable size when using modern analytical control synthesis methods. Second, the *controller reduction* is often necessary to obtain low complexity, hardware/software implementable controllers. The task of application-specific model reduction is an iterative design process on three possible decision strata: 1) the selection of free tuning parameters (e.g. order of reduced model/controller or parameters of frequency weights); 2) the choice of an application-suited reduction method; 3) the choice of application-suited error or performance criteria. For all these decision strata, the control-engineering computation environment ANDECS[1] provides a generic framework for interactively performing iterative computational experiments. In particular, for iterative tuning of free parameters, ANDECS provides frames for different kinds of design modes such as automatic parameter variations, direct parameter search by the designer, or automatic demand-driven optimization (Grübel *et al.*, 1993*c*).

The paper presents the new integrated model-reduction facilities available within ANDECS. The computational basis is a collection of Fortran routines, available in the RASP-MODRED library, implementing the latest algorithmic developments in the domain of computational methods for model reduction (Varga, 1994). The interaction modules provide versatile user interfaces to main computational routines, allowing to solve complex model reduction applications by combining various modules. These modules together with other ANDECS modules for performance evaluation, systems analysis, optimization, simulation, and graphics, form a powerful integrated computational environment for complex model reduction studies.

[1] ANDECS® (ANalysis & DEsign of Controlled Systems) is a trademark of DLR

## 2. METHODOLOGICAL ASPECTS

For the design of low order controllers for high dimensional systems, the following three-steps approach is frequently used in practice.

1. For an $n$-th order system $G_n$ compute an $n'$-th order approximation $G_{n'}$ $(n' < n)$.
2. For the $n'$-th order system $G_{n'}$ design a controller $K_{r'}$ of order $r'$.
3. For $r'$-th order controller $K_{r'}$ compute an $r$-th order approximation $K_r$ $(r < r')$.

**Note.** $G_n$ simultaneously denotes an $n$-th order state space system $(A, B, C, D)$ and the corresponding *transfer-function matrix* (TFM) $G_n(\lambda) = C(\lambda I - A)^{-1}B + D$.

Reasons for the order reduction at step 1 are to ensure the possibility to solve controller design problems at step 2 or to obtain directly lower order controllers at step 2, avoiding the usually more involved controller reduction at step 3. The computed approximation should guarantee a reasonably good fit expressed by small norms of approximation errors, good matching of specific parameters, good agreements of time-responses to various kinds of input signals etc. Desiderata are to preserve basic system properties such as stability, minimum-phase behavior, minimality, etc. It worths to mention the lack of all-purpose methods which can guarantee the fulfillment of all the above requirements.

If the original system $G_n$ is stable, then at step 1 usually one of the following model reduction methods can be used: the *balance and truncate* (B&T) method (Moore, 1981), the *singular perturbation approximation* (SPA) method (Liu and Anderson, 1989), the *Hankel-norm approximation* (HNA) method (Glover, 1974) or the *balanced stochastic truncation* (BST) method (Desai and Pal, 1984). The B&T, SPA and HNA methods can be viewed as methods which try to minimize the absolute approximation error $\|G_n - G_{n'}\|_\infty$. The HNA method produces a solution which at the same time is the optimal solution of the analogous Hankel-norm minimization problem. The BST method is basically a method trying to minimize the relative approximation error $\|G_n^{-1}(G_n - G_{n'})\|_\infty$. All these methods possess computable guaranteed *a priori* bounds for the corresponding approximation errors. Moreover, the resulting reduced models are minimal and stable.

The *frequency-weighted model reduction* (FWMR) can be used in conjunction with several basic methods to enforce better approximations on selected frequencies ranges. This approach tries to minimize the weighted approximation error $\|W_1(G_n - G_{n'})W_2\|_\infty$, where $W_1$ and $W_2$ are the *output* and *input weighting* transfer-function matrices, respectively. The basic FWMR approaches are related to the B&T (Enns, 1984) and HNA (Latham and Anderson, 1985) methods.

If the original system $G_n$ is unstable, then the model reduction can be performed by reduction of only the stable part $[G_n]_-$ from its additive stable/unstable decomposition $G_n = [G_n]_- + [G_n]_+$. An alternative approach, the *coprime factors model reduction* (CFMR) method (Liu and Anderson, 1986), can be used to reduce the stable factors $N_n$ and $M_n$ of a *left* or *right* stable rational coprime factorization of $G_n$ in the forms $G_n = M_n^{-1}N_n$ or $G_n = N_n M_n^{-1}$, respectively. The CFMR method can be used in conjunction with any of the already mentioned methods for reduction of stable systems. Particular coprime factorizations, as the factorization with $M_n$ *inner* or the *normalized coprime factorization* may have additional advantages (Varga, 1993*a*).

The controller reduction at step 3 is necessary if controllers computed at step 2 have too high orders for practical use. The main requirement is to guarantee the closed-loop stability when the reduced controller is used instead of the original one. For controllers resulting from robust synthesis techniques an additional requirement is the preservation of similar good robustness properties. For controller reduction all of the model reduction techniques mentioned previously can be used. The performance and the stability aspects can be explicitly addressed in the formulation of the controller reduction problem. In one of the approaches proposed in (Anderson and Liu, 1989) the controller reduction problem can be translated into a frequency-weighted model reduction problem to minimize $\|W_1(K_{r'} - K_r)W_2\|_\infty$, where the weights have to be chosen as $W_1 = (I + G_{n'}K_{r'})^{-1}G_{n'}$ and $W_2 = (I + G_{n'}K_{r'})^{-1}$. Difficulties to solve this problem may arise because of the usually high order input- and output weighting matrices and of a possibly unstable controller.

## 3. RASP-MODRED LIBRARY

The subroutines of RASP-MODRED cover all computational problems mentioned in the previous section for order reduction of both continuous-time and discrete-time systems. The algorithms implemented in this library are numerically reliable and computationally efficient and represent the latest developments in the field of numerical methods for model reduction. RASP-MODRED is among the first libraries developed by using the new linear algebra package LAPACK (Anderson *et al.*, 1992) and is certainly the first library written in Fortran which provides a rich set of computational facilities for model reduction.

The reduction of stable systems can be performed by using variants of the already mentioned methodologies related to balancing techniques: B&T, SPA, HNA and BST. The algorithms implemented in the core model reduction routines are the recently developed *square-root* and *balancing-free* accuracy enhancing methods developed in (Varga, 1991*a*,*b*; Varga and Fasol, 1993). For the reduction of unstable systems, the available tools for the reduction of stable systems can be used in conjunction with the coprime factor model reduction technique or the additive spectral decomposition approach. Several new algorithms for computing stable coprime factorizations of transfer-function matrices are implemented (Varga, 1993*a*,*c*). For performing frequency-weighted model or controller reductions, tools are provided to compute efficiently and in a numerically reliable way the necessary stable projections (Varga, 1993*b*). Additional tools are available for computing Hankel and $L^2$ norms of transfer-function matrices (Varga, 1992).

In its present state of development, RASP-MODRED consists of 77 routines. About 90 various LAPACK and BLAS routines are called directly or indirectly by the MODRED routines. The implementation of routines has been done according to the RASP/SLICOT mutual compatibility concept described in (Grübel *et al.*, 1993*a*).

## 4. INTEGRATED MODEL REDUCTION FACILITIES

The integrated model reduction facilities are centered around the new ANDECS modules incorporating the main model reduction routines from RASP-MODRED. The model reduction modules from ANDECS can be used independently to perform model reduction or model evaluation tasks, but can be also used in computation sequences within the ANDECS Multi-Objective Programming System (MOPS) (Grübel *et al.*, 1993*b*). Such *integrated* usage of model reduction modules in conjunction with other ANDECS modules for system analysis, simulation, optimization, or graphics, is a superior approach for performing model reduction tasks. Such usage allows: to conduct an automated search for a suitable reduced order model which fulfills some desired requirements; to store and retrieve for post-processing reduced models of different orders and/or obtained by using different methods; to document the performed computations via convenient hierarchical naming conventions and an automatic database storage scheme.

The development of application-specific reduced-order models is a design search process, where both the design object (reduced order model) and the corresponding specifications (model performances) are iteratively adapted in order to yield a suitable compromise between potentially conflicting model reduction goals. The MOPS environment of ANDECS provides facilities to perform systematic parametric search studies based on a multi-criteria goal-attainment approach. This environment also provides facilities to record a whole model reduction history in an automatically evolving database. This allows the comparison of different results by backtracking and thereby supports model assessment over an entire design project.

Figure 1 depicts the MOPS design-logic to solve a parameterized model reduction problem. The basic block **Model Reduction** computes a reduced order approximation P from a high order model or controller. The free parameters in the model reduction method are assembled in the tuning vector T. These parameters, for instance, can be a specified order, a specified bound for approximation error, parameters of input or output frequency weighting transfer functions, etc. The model reduction is performed by using the basic model reduction modules.

The next step is to build an **Evaluation Model** M to be used in computing performance indicators which are specific

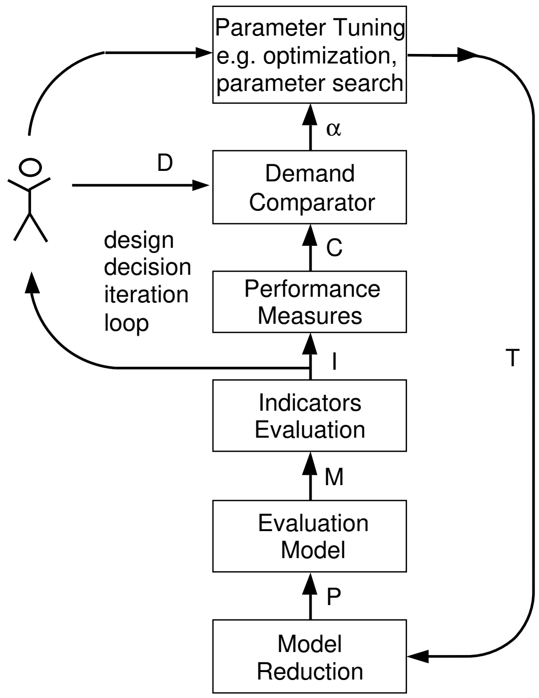


Fig. 1. The ANDECS Model Reduction Machine

to the application area. In the simplest case M is the resulting model P itself. For more involved performance analyses, more complex evaluation models can be used as for instance an error model, a frequency weighted error model or even a closed-loop system model. More complex evaluation models can be generated for simulation purposes by exploiting the ANDECS connecting facilities to external modeling environments as ACSL or Dymola.

The block **Indicators Evaluation** typically performs simulation runs to compute time responses, evaluates frequency responses, or performs specific system analysis tests on the computed reduced order model. The indicators I are transferred to the block **Performance Measures** which transforms them into a performance criteria vector C whose components $c_i, i = 1, \ldots, k$ value the "goodness" of an obtained approximation by, e.g., integrals of output errors for specific inputs, norms of errors in frequency responses, etc.

As a **Demand Comparator** we use the max-function $\alpha = \max\ \{c_i/d_i, i = 1, \ldots, k\}$, where each demand $d_i$ is the particular performance weight corresponding to the criterion $c_i$. D is a vector with components $d_i, i = 1, \ldots, k$. If $\alpha < 1$ then we have C < D, that is

$$\begin{bmatrix} c_1 \\ \vdots \\ c_k \end{bmatrix} \leq \alpha \begin{bmatrix} d_1 \\ \vdots \\ d_k \end{bmatrix} < \begin{bmatrix} d_1 \\ \vdots \\ d_k \end{bmatrix} .$$

A reduced order model is the better, the smaller the criteria vector C is compared to the performance weight vector D. Hence $\alpha = \max\ \{c_i(T)/d_i\}$ should be minimized as a function of the parameters T which specify the desired model reduction. This is the purpose of the block **Parameter Tuning** which adjusts T by help of suitable techniques. For instance, the adjustment of vector T can be the result of a parametric optimization of the form

$$\min\ \alpha(T)$$

subject to constraints of the form

$$c_i \leq \overline{c}_i\,; \quad \underline{T}_j \leq T_j \leq \overline{T}_j\ .$$

T can be also adjusted on the basis of systematic parameter variation strategies conducted over a range of possible orders or over specified ranges of values of frequency weighting parameters.

Within the design environment ANDECS, the visualization environment VISTA (Visualization and Interactive Steering for Task Activation) (Grübel et al., 1993b) is available for online interaction and result visualization. Multiple diagrams, e.g. Bode/Nyquist frequency response diagrams, time responses, etc. may be used to visualize the indicators assessing the quality of model reduction. Special diagrams such as representations of the performance vector in parallel coordinates can be used to visualize the conflicts among multiple criteria. These visual aids help the designer in making locally consistent decisions in directing the search process towards a satisfactory compromise solution.

**Example.** We present in brief the integration of an interactive module for frequency-weighted model reduction (FWMR) into the ANDECS model reduction frame depicted in Fig. 1. The block **Model Reduction** calls two ANDECS modules to solve a FWMR problem by employing the methodology of (Latham and Anderson, 1985). In the FWMR problem formulated in Section 2 we assume that $G_n$ is a $p \times m$ stable TFM of an $n$-th order system and $W_1$ and $W_2$ are respectively, $p \times p$ and $m \times m$ invertible TFMs having only unstable poles and zeros. Then the following approach can be used to solve the FWMR problem:

1. Compute $G_1$, the $n$th order stable projection of $W_1 G W_2$.
2. Determine $G_{1n'}$, an $n'$th order approximation of $G_1$ by using a model reduction method suitable for stable systems.
3. Compute $G_{n'}$ the $n'$th order stable projection of $W_1^{-1} G_{1n'} W_2^{-1}$.

A set T of 7 free parameters which specifies the model reduction to be performed is defined as follows. One of the parameters is the desired order $n'$. The remaining parameters belong to the first order weighting matrices

$$W_1 = K_1 \frac{s + a_1}{s + b_1} I_p\ , W_2 = K_2 \frac{s + a_2}{s + b_2} I_m\ ,$$

with $a_1 < 0, b_1 < 0, a_2 < 0, b_2 < 0$. Then we have $T = \{n', K_1, a_1, b_1, K_2, a_2, b_2\}$.

As **Evaluation Model** M we can use besides the reduced order model $G_{n'}$, also the error model $E = G_n - G_{n'}$. As a time domain indicator for the performed model reduction we can use the error between the step responses of the original and reduced systems. As a frequency domain indicator we can use the norm of the approximation error $E$. Qualitative indicators are: stability, minimum/non-minimum phase characteristic, poles/zeros excess.

The computed indicators serve to assemble a performance vector C. In our case a performance vector with two components $c_1(T)$ and $c_2(T)$ is appropriate. Define

$$c_1(T) = \int_{t_1}^{t_2} \|y_n(t) - y_{n'}(t)\|^2\, dt\,,$$

where $y_n(t)$ and $y_{n'}(t)$ are the step responses of the original and of the reduced order systems, respectively, and

$$c_2(T) = \max_{\omega_1 \le \omega \le \omega_2} \quad \| G_n(j\omega) - G_{n'}(j\omega) \| ,$$

where $G_n(j\omega)$ and $G_{n'}(j\omega)$ are the frequency responses of the original and of the reduced order systems, respectively. The interval $[\omega_1, \omega_2]$ should be chosen consistent with the frequency weights used to enforce better approximation on certain frequency ranges.

The elements of the demand vector D in the **Demand Comparator** express the relative importance of the chosen performance measures to yield a satisfactory approximation. In our case the choice of a smaller value for $d_2$ could reflect the emphasis on a good approximation on a certain frequency region.

The **Parameter Tuning** can be done by a systematic search procedure over different possible orders combined with an optimization procedure which modifies the rest of parameters of the vector T.

This example illustrates one of many possible ways to integrate a model reduction module into a multi-objective "best-compromise" parametric search. The main advantage of this approach is its flexibility to combine different tools, to define different performance measures, or alternative parameter tuning strategies.

## 5. CONCLUSIONS

In this paper we presented the recently developed integrated model reduction facilities within ANDECS. The superiority of the ANDECS model reduction tools over similar tools available in commercial packages resides in the functional richness to support different methodological approaches, in the usage of better numerical algorithms and of more robust numerical software, in the advanced aids for a flexible integrated model reduction approach and in the advanced data storage and results-backtracking facilities. The ANDECS suite for model reduction is seen as one of the most powerful and versatile software tools capable to perform complex model and controller reduction tasks. Further developments of ANDECS envisage the automation of solving complex problems by the help of expert modules based on a model-reduction methodological knowledge.

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